# H31B-0865: Modeling the Effects of Variable Recharge and Conduit Geometry on Flow Through Karstic Aquifers

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## Summary

Roughly 20-25% of the global population relies on groundwater from karstic aquifers, which contain heterogeneous conduit systems that can carry water and contaminants at velocities of meters per second. The difficulty of modeling these systems results from a poor understanding of the geometry of the conduits networks. The storm response of a karstic aquifer can be used to probe the aquifer for information about its geometry. In this work we explore three fundamental questions regarding storm response of karstic systems:

**1) What can be determined about the geometry of a karstic system given only the inputs and outputs?**

**2) What must be known about the geometry to produce a predictive model of the response of a karstic aquifer?**

**3) To what extent is the discharge a function of recharge versus geometry?**

## I. Previous Work

There is a long history of spring hydrograph analysis of karstic aquifers (see for example Ford and Williams, 2008). The vast majority of this work has focused on the recession periods of hydrographs following storm events. **Recession curves** have been used to decompose the aquifer response into saturated (base flow) and unsaturated (quick-flow) components (Mangin, 1975). Another approach is to derive **kernel functions** for a given aquifer such that when convolved with the rainfall they produce the observed response hydrograph (Dreiss, 1983). However, for lumped system modeling, non-linear, non-stationary systems are required (Anaya and Wanakule, 1993) and little is known about the correlation between lumped system parameters and the geometry of the aquifer.

Some authors have made estimates of various **geometrical parameters** of the aquifer via the analysis of hydro- and chemographs. Geometrical parameters estimated include conduit volume (Ashton, 1966; Birk, Liedl, and Sauter, 2006), aquifer heterogeneity (Mangin, 1975), and the ratio of conduit surface area over volume and flow path length (Grasso and Jeannin, 2003).

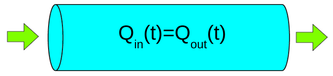
Despite this large body of work, many questions remain. Little attempt has been made to fully explore the parameter space in order to estimate the certainty with which various geometrical parameters can be determined. Furthermore, most current approaches only analyze the system outputs and do not use information about the input into the system. The reason for this is pragmatic (often we don't know the input function). However, in some circumstances the output is primarily a function of the input and not of the system geometry (see section III, and Figure 3). This study uses a more general framework to begin addressing these issues.

## II. Elements of a Karstic Aquifer

A karstic aquifer is composed primarily of two elements: **conduits,** and a **porous matrix** (which may include significant fracture porosity). At any point in time the conduits may behave in three different ways with respect to how they modify a flood pulse. They can contain **full-pipe flow**, **open channel flow**, or act as a **reservoir**. In isolation, each of these modes will modify a storm pulse in a particular way.

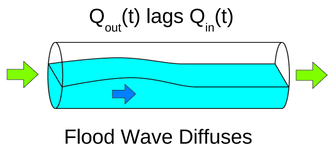
## II-A. Full-Pipes

A full conduit has no effect on the shape of a storm pulse, because **at all times the input equals the output.** For karst conduits inertia is negligible, and it is a good approximation to assume that all changes in head at the upstream end of the pipe are instantaneously felt at the downstream end. The flow at a given instant is a function of the boundary conditions and the frictional losses along the pipe.



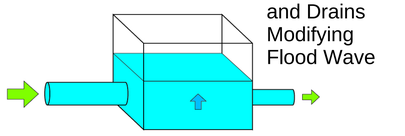

## II-B. Open Channels

In an open channel a **flood wave will diffuse** such that the **peak discharge decreases** with distance and the **width of the wave increases**. Additionally the wave is skewed toward the recession tail. Open channels also introduce a time lag such that the spring does not respond instantaneously to input. Both the diffusion and time lag are a function of the channel's cross section and length, such that longer and wider channels produce more diffusion and lag (Figure 1).




## II-C. Reservoirs

Reservoirs act upstream of a constriction. When the input exceeds the flow through the constriction the reservoir will begin to fill. The reservoir can drastically modify the shape of the incoming flood pulse. As for the open channel, the pulse will be dispersed. However, the output flood wave will contain **filling** and **draining** portions that will in general have different slopes. In many cases the output will largely be a function of the geometry of the reservoir/constriction, bearing little resemblance to the input (Figure 2).



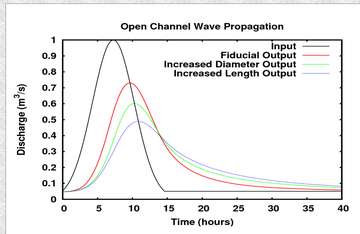




**Figure 1 –** Propagation of a flood wave down an open channel simulated by SWMM5. The solid black line depicts the input function. All other lines are channel outputs. The fiducial channel (solid red line) has a diameter of 5 meters and a length of 3 km. Also shown are a 3 km channel with a diameter of 10 meters (dashed green) and a 5 km channel with a 5 meter diameter (dotted blue). Larger diameters and longer channels produce more dispersion and time lag.

## II-D. Porous Matrix

For many karstic aquifers, the porous matrix will only affect the tail end of the recession curve. However, for extremely porous aquifers, as found in some young carbonate platforms, the porous matrix may significantly affect the pulse. Flow into the matrix during the rising limb of the storm will **decrease the peak flow** of the wave. Ultimately this water will flow back into the conduit during recession thus **skewing the pulse toward the recession limb**.

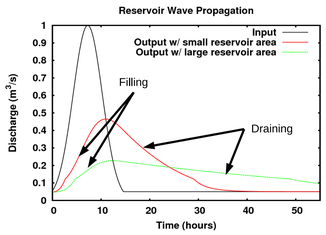



**Figure 2 –** Propagation of a flood wave through a reservoir simulated by SWMM5. The solid black line shows the input. For both cases the reservoir is drained through a conduit 1 meter in diameter and 10 meters long, and the reservoir walls are assumed to be parallel. The two cases depict reservoirs with different surface areas: 3,000 $m^2$ (red line), and 15,000 $m^2$ (green line).

## III. Varying Recharge

The response of a karstic system will vary as a function of recharge. One example of this is the presence of thresholds in the system. Once a certain flow is surpassed a reservoir may begin to fill or an overflow route may become accessible. Unfortunately, these thresholds cannot be predicted before they are observed.

The rate of change of the input is also an important factor. When the **input changes rapidly** it throws the aquifer furthest from equilibrium so that the **geometry of the system is important** in determining the output. However, if the **input changes slowly** compared to the response time of the system then the **output will look very similar to the input** (Figure 3). This fact is particularly important when assessing recession curves, which are used to characterize the slow-flow portion of the system. Similar recession tails are seen in surface streams as the result of inflow from the soil. Therefore, in some cases, the recession curve of a karst spring may provide no information about the aquifer itself but rather about a receding input.

Finally, analyzing system responses to storms with various intensities and durations provides additional information about the system. In inverse modeling this helps to reduce problems with solution uniqueness (see section IV) because it is more difficult to produce two systems with different geometries that will respond equivalently to a variety of inputs than it is to produce two systems with different geometries that respond equivalently to a single storm.

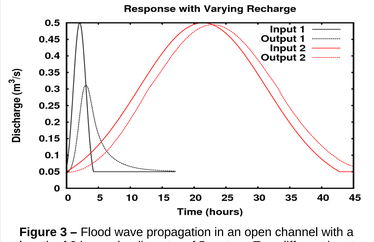



**Figure 3 –** Flood wave propagation in an open channel with a length of 2 km and a diameter of 5 meters. Two different inputs and outputs are shown. Both inputs are gaussians with a peak of 0.5 $m^3$/s. One input has a width of σ=1.0 hours (black) and the other has a width of σ=10 hours (red). The channel only modifies the pulse if the rate of change of input is slow compared to the response time of the system. For the slowly changing input, there is a time lag at the output but no change in the shape of the wave.

## IV. Inverse Modeling

To address the ultimate goals of this project we will conduct an extensive exploration of the parameter space of karstic aquifer properties. One of the chief difficulties is that the **model solution for an aquifer response is highly non-unique**. For example, the discharge response of a single conduit is indistinguishable from the response of two conduits in parallel. Similarly, greater dispersion and time lag in an open channel can be produced by increasing the length or the width of the channel. **Some of these uniqueness problems can be resolved by analyzing chemo- and thermographs** because the volumes and travel times of the suggested equivalent flow systems are not the same. Volume and travel times can be estimated using chemical and thermal responses (Ashton 1966).

### Modeling Approach

We will construct a number of idealized karstic aquifers with increasing complexity and subject these models to a series of storm events. The models are built using the Storm Water Management Model (SWMM5). This code, designed for simulating flow through sewer drainage systems, allows one to simulate flow through a system of conduits, channels, and reservoirs. The model solves the St. Venant equations for open channel flow. It also will calculate exchange with groundwater, analogous to karst's porous matrix. For simple systems the model can be executed in only a few seconds, making it possible to explore parameter space efficiently.

For each idealized aquifer we will assume we know nothing of the geometry and then calculate numerous inverse models, attempting to match the storm responses. Geometrical parameters for each inverse model will be produced using Monte Carlo Markov Chains (MCMC, see Sambridge and Mosegaard 2002 for a review), allowing us to simultaneously search for an optimum parameter set as well as characterize the parameter space. The distribution of models from the MCMC can be used to determine the extent of the uniqueness problem, and will allow us to calculate the stiff and sloppy directions in the parameter space.

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