What's in a Spring Hydrograph?

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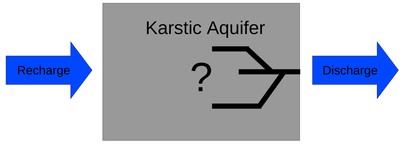
Summary

Many previous studies have used spring hydrographs to probe the properties of a karst aquifer. However, **little work has been done on the influence of the aquifer recharge on the discharge hydrograph.** In order to study the manner in which karst conduits modify input flood hydrographs, we develop a simple conceptual model of flood pulse propagation through karst conduits.

First, we identify three fundamental behaviors of karst conduits in response to flood pulses: full pipe, open channel, and reservoir/constriction response. We analytically **derive characteristic response times** for each type of conduit flow and **define a dimensionless number that is the ratio of the conduit response time to the characteristic time of a recharge event.** This number characterizes the response of a given conduit segment for a given flood event, **dividing responses into geometry-dominated and recharge-dominated regimes**, in which the discharge hydrograph is either modified or resembles the recharge hydrograph, respectively.

Given typical parameters of karst conduits, we conclude that many, if not most, karst conduits are unlikely to modify the input flood hydrographs. This finding provides important constraints on the amount of information that can be derived via inverse modeling of spring hydrographs. In particular, **we show that for many karst conduits the discharge hydrograph is nearly identical to the recharge hydrograph and thus contains little information about the conduit geometry.**

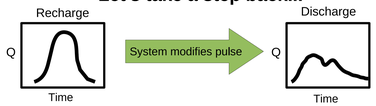
The central problem in karst hydrology?



Conduits largely determine the flow and transport through the system, but in most cases we have incomplete information about the conduit network.

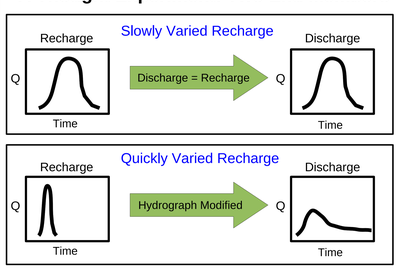
A traditional approach: analyze spring hydrographs to derive geometrical information (e.g. recession analysis).

Let's take a step back...



and ask fundamental question:
What is the effect of recharge on the discharge hydrograph?

A Thought Experiment: Two End Members



How do we define "slowly" and "quickly" varied recharge?

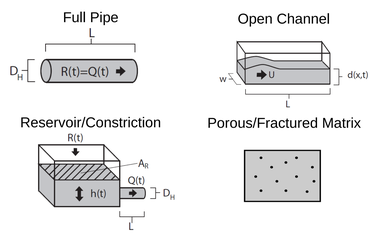
Using a dimensionless number:

$$\gamma = \frac{System\ Response\ Time}{Recharge\ Time}$$

According to the thought experiment above:
high gamma should result in significant modification of the recharge by the system
and
low gamma should result in little modification of the recharge.

Elements of a Karstic Aquifer

A karstic aquifer is composed primarily of two types of elements: **conduits,** and a **porous/fractured matrix**. The conduits may behave in three different ways with respect to how they modify a flood pulse. They can behave as a **full-pipe**, **open channel**, or as a **reservoir/constriction**. Each of these response modes will modify a storm pulse in a particular way.



Characteristic Element Response Times

Full Pipe

1D Euler Equation for pipe flow gives a finite momentum response time

$$\frac{L}{g}\frac{dV}{dt} + \frac{fLV|V|}{D_H 2g} = h_{in},$$

Typical response time is only seconds!

$$\tau_{mom} \sim \frac{D_H}{fV_f}.$$

Open Channel

Kinematic Wave approximation gives wave velocity

$$U \sim \frac{3}{2}V.$$

Response Time

$$\tau_{open} = \frac{L}{U},$$

Reservoir/Constriction

Continuity Equation

$$\frac{dh}{dt} = \frac{R(t) - Q(t)}{A_R},$$

Darcy-Weisbach Equation

$$Q(t) = A_c\sqrt{\frac{2g}{(1 + fL/D_H)}h(t)},$$

Response Time

$$\tau_{res} \sim \frac{A_R C_f R_{peak}}{2gA_c^2}.$$

Matrix

Recession Coefficient

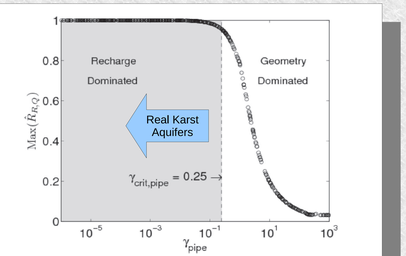
$$\alpha = 1/\tau_{hydraulic}.$$

$$\tau_{hydraulic} \sim \frac{\phi L^2}{Kd},$$

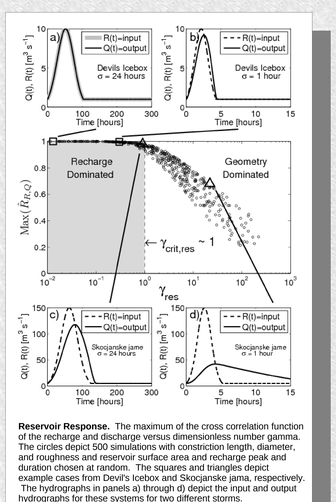
This time scale is typically significantly longer than the time scale of storms.

Testing the thought experiment

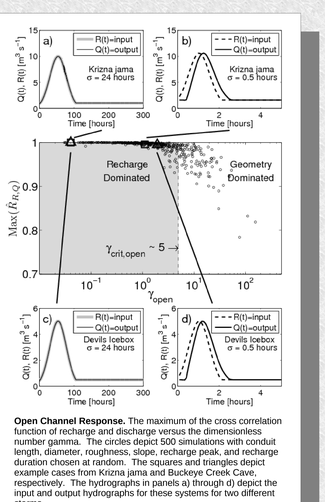
- Run numerical simulations of flood response of simple single-element systems and compare input and output hydrographs
- Sample parameter space of each element type randomly within the range of realistic physical values
- Input gaussian recharge curves with a width σ that can be used for characteristic recharge time scale. Sample many values of σ, peak recharge, and base recharge
- Full pipe and reservoir calculated via Runga-Kutta integration, and open channel flow simulated using the Storm Water Management Model (SWMM) developed by the EPA
- Use cross-correlation of the recharge and discharge hydrographs to quantify the extent of modification by the system



Full Pipe Response. The maximum of the cross correlation function of the recharge and discharge versus the dimensionless number, gamma. The circles depict 500 simulations with conduit diameter, Darcy friction factor, recharge peak, and recharge duration chosen at random. The aquifer response can be divided into two regimes: a recharge-dominated regime at low gamma and a geometry-dominated regime at high gamma. For almost all natural karst conduits and realistic input hydrographs gamma<< 0.25.



Reservoir Response. The maximum of the cross correlation function of the recharge and discharge versus dimensionless number gamma. The circles depict 500 simulations with constriction length, diameter, and roughness and reservoir surface area and recharge peak and duration chosen at random. The squares and triangles depict example cases from Devil's Icebox and Skocjanske jama, respectively. The hydrographs in panels a) through d) depict the input and output hydrographs for these systems for two different storms.



Open Channel Response. The maximum of the cross correlation function of recharge and discharge versus the dimensionless number gamma. The circles depict 500 simulations with conduit length, diameter, roughness, slope, recharge peak, and recharge duration chosen at random. The squares and triangles depict example cases from Krizna jama and Buckeye Creek Cave, respectively. The hydrographs in panels a) through d) depict the input and output hydrographs for these systems for two different storms.

Conclusions

- Full pipes and open channels from natural karstic aquifers are unlikely to modify the hydrograph of a storm pulse propagating through them.
- Reservoir/Constrictions can modify storm pulses, but many do not.
- The matrix component will typically modify storm pulses, whereas single conduit segments only do so in special cases.
- This places strong limitations on inverse modeling of conduit network geometry from hydrographs, since in many cases the hydrograph contains no information about the conduits.

Future Work

- Complex systems of aquifer elements can produce many more effects, including the addition of multiple inputs, transition from one element type to another, overflow, and backflow. In future work we will attempt to extend the simple response time model described here into more complex systems.
- This work used a discrete characterization of recharge and response. It may be possible to develop a technique for continuous characterization, perhaps using Fourier or wavelet analysis.

In Press:

This work is described in detail in:
Covington, M. D., C. M. Wicks, and M. O. Saar (2009), *A Dimensionless Number Describing the Effects of Recharge and Geometry on Discharge from Simple Karstic Aquifers*, **Water Resour. Res.**, doi:10.1029/2009WR008004, in press. (accepted 13 August 2009).

It can currently be accessed via the WRR in press page: http://www.agu.org/journals/pip/wr/2009WR008004-pip.pdf